Multi-objective optimization of wind-excited structures

I. Venanzi*, A.L. Materazzi

University of Perugia, Perugia, Italy

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Abstract

A procedure for the optimization of wind-excited structures is proposed. It is based on the simulated annealing algorithm combined with the dynamic analysis of the response either in the frequency or in the time domain. When the step-by-step dynamic analysis is used, it can handle the case of flexible structures, like masts and lattice towers, whose geometric non-linear effects cannot be neglected. The procedure allows for multiple variables and objectives. The proposed method is used to optimize the configuration of a cable-stayed mast subjected to turbulent wind loading. The results showed that the algorithm is reasonably independent of the first guess configuration and is effective in avoiding local minima. Its wide field of applicability and ease of implementation make the proposed algorithm a powerful design tool for structural engineers.

Keywords: Optimization; Wind loads; Simulated annealing; Dynamic analysis; Non-linearity

1. Introduction

The optimal design of structures aimed at fulfilling safety and serviceability requirements may be a complex task, especially when the dominant loading excites the dynamic response, whose effects cannot be neglected in the optimization process. A typical case is that of flexible structures exposed to turbulent wind.

In recent years the development of optimization techniques has supplied the designers with interesting tools to solve this difficult problem.

As it is well known, optimization problems can be dealt with by several well-established techniques when the objective function can be expressed in closed form, i.e. there exists a linear or non-linear relationship between the design variables and the target to be minimized.

In structural dynamics problems, the analytical relationship between the target functions involving the unsteady response and the design variables can be found only in a few cases, unless resorting to approximations. This is the case of the wind excited response of structures, which can be expressed in closed form through the assumption of Gaussian wind pressure, linear structural behavior and statistical independence of the modal responses. Moreover, even if the objective function can be evaluated analytically, it may present several local minima, and therefore a global optimization technique, capable of finding the global minimum of the problem, escaping from the local minima, is needed.

To overcome these difficulties, a simulation procedure combined with a global optimization technique can be used. In this way, it is possible to take advantage of the global algorithms’ capability to avoid local minima and of the simulation procedures’ capability to correctly represent the stochastic nature of the physical phenomenon involved. This can be achieved through the evaluation of the objective function in correspondence of a large number of realizations of the response process.

Engineers have been interested in the optimization of structures exposed to wind load for many years. In 1975 Bell & Brown [1] proposed to optimize guyed towers through a method based on the Powell search and Branch-and-Bound sub-optimization routines that generate locally optimum designs in terms of each design variable considered separately. In 1990 Thornton et al. [2] developed a computer program for the weight optimization of tall structures under deflection control. In both cases the wind load was modeled using equivalent static forces.

Afterwards, many efforts were made to implement optimization strategies to maximize the dynamic systems

A modified iterated simulated annealing method with sensitivity analysis and automatic reduction of the feasible region was proposed in 1997 by Pantelides and Tzan [5] for the optimal design of structures under dynamic constraints. It gave better performance than standard mathematical programming methods.

Other stochastic algorithms such as random search methods, clustering methods and genetic algorithms currently represent a powerful tool to solve a large variety of optimization problems [6].

In the present paper the Simulated Annealing algorithm was chosen, among the global optimization techniques, for its effectiveness and easiness of implementation. The strength of the procedure mainly lies in its adaptability to different design situations. Therefore, it is also suitable to solve multiple objective optimization problems, which occur in most practical cases.

In the following sections the use of a new optimization method, based on the Simulated Annealing Algorithm, is proposed and explained in detail. Then, numerical applications to a case study, consisting in a cable-stayed mast subjected to turbulent wind, are presented and critically discussed.

2. The optimization technique

2.1. The Simulated Annealing algorithm

The Simulated Annealing algorithm is used in combinatorial optimization problems and owes its name to the similitude with the process of melting and subsequent slow crystallization of a metal. During the annealing, the metal particles slow down from the free and rapid movement typical of the liquid phase and set in pure crystals, reaching the minimum energy state.

The probability distribution function of the energy \( E \), assumed depending only on the temperature \( T \), follows the Boltzmann distribution:

\[
P(E = \bar{E}) = \frac{1}{Z(T)} \cdot \exp\left(-\frac{\bar{E}}{k_b \cdot T}\right)
\]

where \( Z(T) \) is a suitable normalization factor and \( k_b \) is the Boltzmann constant. As the temperature reduces, the Boltzmann distribution concentrates on the states with lower energy, and finally, when the temperature approaches zero, only the minimum energy states have a non-null probability of occurrence. This physical process was mathematically modeled by Metropolis et al. [7], who proposed an algorithm to simulate a series of system’s states towards the thermal equilibrium.

The Simulated Annealing algorithm is composed by a sequence of Metropolis algorithms calculated in correspondence of decreasing values of a suitable control parameter representing the temperature [8].

Given a succession of options, characterized by different levels of the energy \( E_i \), the algorithm generates a series of configurations with energy \( E_{i+1} \), having probability proportional to \( \exp\left(-\frac{(E_{i+1} - E_i)}{T}\right) \). This means that the algorithm proceeds towards a thermal equilibrium state, always following a downhill path, but sometimes taking an uphill direction. This behavior enables the procedure to escape from local minima.

It can be demonstrated that under proper hypotheses on the convergence parameters, corresponding to a sufficiently slow cooling, the succession of the system’s states converges to the global minimum [9].

The implementation of the Simulated Annealing algorithm requires the definition of the following data:

- a large set of possible configurations, which represents the possible states of the physical system;
- an objective function to minimize, corresponding to the internal energy of the system;
- a control parameter and a cooling schedule, which represent the temperature and its rate of decrease, respectively;
- a generator of random changes of the system configuration that simulates the path of the system towards the lower energy state.

The algorithm consists essentially of two main loops. The external one controls the temperature decrease and the internal one is the implementation of the Metropolis algorithm, which provides the required system perturbation and checks for the acceptability of the new configuration. In Fig. 1 the block diagram of the Simulated Annealing algorithm is shown.

Preliminarily, the initial temperature \( T_0 \), the final temperature \( T_f \), the coefficient of reduction of the temperature \( k \), and the total number \( N \) of configurations that must be accepted in each internal loop are set. The first set of the design variables \( x_0 \) is initialized to first guess values and the number of accepted configurations \( n \) is set equal to zero. Then, the internal loop begins with the generation of a new set of the design variables \( x_i \). The corresponding value of the objective function \( f(x_i) \) is computed. If the constraints are fulfilled, the penalty function \( P(x_i) \) is set to zero; otherwise it is set to a constant value sufficiently high to force discarding the configuration. The function \( F(x_i) \) is then evaluated as the sum of \( f(x_i) \) and \( P(x_i) \). The difference \( \Delta F \) between the function \( F(x_i) \) and \( F(x_i+1) \) evaluated at the previous step is computed. If \( \Delta F \) satisfies the conditions:

\[
\Delta F < 0 \quad \text{or} \quad \text{rand}(0,1) < \exp\left(-\frac{\Delta F}{T}\right)
\]

which represent the acceptance criterion, the configuration is accepted otherwise it is discarded. In the first case the number of the accepted configurations \( n \) is updated and the internal loop is repeated until \( n \) equals \( N \). Then, on the external loop, the temperature \( T \) is decreased and the procedure is repeated with the updated temperature. The external loop ends when the condition \( T = T_f \) is reached.

To make the optimization procedure usable for problems with continuous variables some modifications in the generation

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[Note: The document seems to be a continuation of a scientific paper discussing optimization techniques specifically the Simulated Annealing algorithm and its implementation. It covers the theoretical background, the algorithm's structure, and its application to structural optimization problems including natural frequencies and vibration levels. The text is interspersed with mathematical formulas and algorithms related to the Metropolis step, the definition of the acceptance criterion, and the overall procedure of the algorithm with examples of its application. The document emphasizes the adaptability and effectiveness of the Simulated Annealing algorithm for solving complex optimization problems.]
of random changes are needed. It is not possible to provide the algorithm with a set of possible configurations, but only the first guess design variables can be supplied.

In the case of continuous systems the generation of the random perturbation of the design variables is an important issue. In fact, a good generator should always provide a trial point in the downward direction to increase the efficiency of the algorithm but also let the algorithm accept sometimes a trial point in the upward direction to escape local minima. The procedure proposed in the present paper uses the Downhill Simplex Method to create the random sequence. In the neighborhood of the trial point a simplex is generated and the values of the objective function at its edges are calculated. Then, the simplex is subjected to proper reflections, expansions and contractions. In each vertex of the modified simplexes the acceptance criterion is checked.

In this continuous design variables case, a non-classical Metropolis acceptance criterion is used. A positive, logarithmically distributed, random variable is added to the value of the objective function in correspondence of each vertex and an analogous random variable is subtracted to the trial value of the objective function. In this way, the configurations in the downward direction are always accepted but sometimes a set of random variables in the upward direction may be accepted too. This allows the algorithm to escape from local minima.

2.2. The multiple objective function

Optimization problems typically involve multiple and often competing objectives. The solution of such problems is not, in general, a unique optimal solution but a set of compromise solutions, known as Pareto-optimal solutions. Each one of these solutions is optimal in the sense that no improvement can be achieved in one objective component without leading to degradation in at least one of the remaining components. Therefore, the primary goal of a multi-objective optimization problem is, unlike that of a single objective optimization, to find several Pareto-optimal solutions to show the precise trade-off information among the competing objectives.

The multi-objective optimization method chosen in the procedure proposed herein is the simple weighting technique. Among the multiple objective methods, it is conceptually the simplest and generates effective solutions every time the objective space \( f(\Omega) \), the projection of the feasible decision space \( \Omega \) into the objective space, is convex and some compensation between the objectives is allowed.

Considering an optimization problem with two competing targets \( f_1(x) \) and \( f_2(x) \), the method consists in solving, for each weighting coefficients set \((\alpha_1, \alpha_2)\), the following single objective optimization problem:

\[
\min f(x) = \alpha_1 f_1(x) + \alpha_2 f_2(x) \quad \text{s.t.: } x \in \Omega.
\]  

In a single objective mathematical programme, dividing the objective by a positive real number does not change its optimum. If \( \alpha_1 > 0 \), dividing the target function by \( \alpha_1 \) and setting \( \alpha = \alpha_2/\alpha_1 \), leads to the following equivalent problem:

\[
\min f(x) = f_1(x) + \alpha f_2(x) \quad \text{s.t.: } x \in \Omega.
\]  

Assuming \( \Omega \) as convex, for a given \( \alpha \), the optimal solution of the problem (4) is an efficient solution of the problem (3). Given different values for \( \alpha \), many optimal solutions can be generated. Than, the best compromise solution may be chosen on the basis of trade-off considerations.

2.3. Implementation of the procedure

In the proposed procedure several structural analysis methods can be implemented, depending on the problem under investigation. If the analyzed structure is subjected to a static loading, the target function will be determined by static analysis of the response. If the structure is subjected to dynamic loading, the objective function is evaluated using linear or non-linear step-by-step analysis or random dynamic analysis, as required by the dynamic system.
As the wind velocity is modeled as a stochastic process, in order to obtain a reliable solution of the optimization problem, the procedure may be repeated many times following a Monte Carlo scheme, modifying, in each run, the initial seed used by the random number generating routine. The statistical analysis of the results leads to the evaluation of the probabilistic structure of the response.

The multi-objective function is evaluated at each step, in correspondence of different trial values of the design variables.

The problem’s constraints are considered through suitable penalty functions by adding a penalty term to the objective function if the constraints are not satisfied. In this case the configuration is automatically discarded. The penalty function approach, using a feasible design only strategy, is suitable for problems with a jointed feasible design space. The case of disjointed feasible domains is not considered in this study as it seldom occurs in structural engineering problems which this study is devoted to.

3. Wind loading and structural response

3.1. Wind load modeling

The time-variant air velocity acting at a point \( P \) of the structure exposed to turbulent wind is composed by a mean and a fluctuating part as follows:

\[
V_P(t) = \bar{V}_P + \tilde{V}_P(t).
\]

The mean velocity \( \bar{V}_P \) is assumed to vary along the height above the ground in the atmospheric boundary layer following the logarithmic law:

\[
\bar{V}_P(z) = K_r \cdot V_{ref} \cdot \ln \left( \frac{z}{z_0} \right)
\]

where \( K_r \) and \( z_0 \) are related to the site roughness and \( V_{ref} \) is the reference wind velocity.

The unsteady component of the wind velocity \( \tilde{V}_P(t) \) is modeled as a null-mean weakly-homogeneous Gaussian random process, whose probabilistic structure is completely defined by its power spectral density function \( S_\nu(P, n) \). Considering the wind acting upon any couple of points \( P \) and \( P' \), the process of the wind velocity fluctuates multicorrelated and its statistical properties are described by the \( 2 \times 2 \) power spectral density matrix \( S_{\nu}(n) \). The main diagonal of \( S_{\nu}(n) \) contains the auto-spectra \( S_n(P, n) \) and \( S_n(P', n) \), while the cross-spectra are commonly expressed as a function of the auto-spectra and of a suitable coherence function \( \text{Coh}(P, P', n) \) depending on the points’ mutual distance and on the frequency \( n \) as:

\[
S_n(P, P', n) = \sqrt{S_n(P, n) \cdot S_n(P', n) \cdot \text{Coh}(P, P', n)}.
\]

In the present study the following expressions are used for the power spectrum \( S_n(P, n) \) and the coherence function \( \text{Coh}(P, P', n) \):

\[
\frac{n \cdot S_n(P, n)}{\sigma_v^2} = 6.86 \left( \frac{f L_v}{z(P)} \right) \left( 1 + 10.302 \frac{f L_v}{z(P)} \right)^{-5/3}
\]

\[
\text{Coh}(P, P', n) = \exp \left\{ -2n \left[ \frac{C_2^2(z(P) - z(P'))^2}{V(z(P)) + V(z(P'))} \right] \right\}
\]

proposed respectively by Solari [10] and by Davenport [11].

The power spectral density of the wind load may be derived by assuming, as is usually done, the quasi-steady relationship between mean square pressure fluctuations and mean square longitudinal velocity fluctuations.

3.2. Frequency domain analysis

In the frequency domain the structural response is obtained by modal superposition.

The power spectral density matrix of the response is given by the following expression:

\[
S_{F}(\omega) = H^*(\omega) S_{F}(\omega) H(\omega)
\]

where \( S_{F}(\omega) \) is the power spectral density matrix of the applied wind load, \( H(\omega) \) is the matrix of the frequency response functions and \( H^*(\omega) \) is its complex conjugate.

The maximum displacements are evaluated as:

\[
x_{\max} = \bar{x} + g_i \sigma_i
\]

where \( \bar{x} \) is the mean response, due to the mean wind load, \( \sigma_i \) is the variance of the response process \( x(t) \) and \( g_i \) is the peak factor given by:

\[
g_i = \sqrt{2 \ln (v_s T)} + \frac{0.5772}{\sqrt{2 \ln (v_s T)}}
\]

in which \( v_s = \frac{1}{2\pi \Delta \omega} \) is the i-th component of the expected frequencies vector for \( x(t) \) and \( \sigma_{vi} \) is the i-th component of the variance of the derived process \( \dot{x}(t) \).

3.3. Time domain analysis

In the time domain approach, a realization of the uni-dimensional multi-variate wind velocity process is artificially generated and the corresponding wind load time histories are applied to the joints of the discretized structure, in each trial configuration.

The simulation of the stochastic process representing the wind velocity is performed using a well established technique [12] as follows:

\[
v_i(t) = \sum_{m=1}^{N} \sum_{i=1}^{i} \left[ H_{jm}(\omega_i) \right] \sqrt{2\Delta \omega} \cos \left( \omega_i t + \theta_{jm}(\omega_i) + \phi_{mi} \right)
\]

where:

\[
\theta_{jm}(\omega_i) = \tan^{-1} \left( \frac{\text{Im} H_{jm}(\omega_i)}{\text{Re} H_{jm}(\omega_i)} \right);
\]

\( H_{jm}(\omega_i) \) is the matrix of the response functions;

\( N \) is the number of intervals along the i-th axis of the frequency domain;

\( \Delta \omega \) is the frequency step.

\[\text{(9)}\]
where \( x_i \) vectors respectively, \( F \) is the load vector, and \( M, C \) and \( K \) are the mass, damping and stiffness matrices.

The geometric non-linearity is considered in the procedure by updating the stiffness matrix. For a generic frame member it is obtained as the sum of the linear stiffness matrix and the geometric stiffness matrix as follows:

\[
\Delta t \frac{\ddot{x}}{2} + \Delta t \frac{\dot{x}}{2} + \frac{1}{2} \Delta t^2 \frac{\dot{x}}{2} = F
\]

\[
\Delta t \frac{\ddot{x}}{2} + \Delta t \frac{\dot{x}}{2} + \frac{1}{2} \Delta t^2 = F
\]

where \( x, \dot{x}, \ddot{x} \) are the displacements, velocities and accelerations vectors respectively, \( F \) is the load vector, and \( M, C \) and \( K \) are the mass, damping and stiffness matrices.

The geometric non-linearity is considered in the procedure by updating the stiffness matrix. For a generic frame member it is obtained as the sum of the linear stiffness matrix and the geometric stiffness matrix as follows:

\[
[K] = [K_{lin}] + \left[ \text{iter} K_{geom} \right]
\]

in which:

\[
[K_{lin}] = \begin{bmatrix}
\frac{E A}{L} & 12EJ & SIMM \\
0 & \frac{6EJ}{L} & 4Ej \\
0 & 0 & \frac{E A}{L}
\end{bmatrix}
\]

\[
[K_{geom}] = \left[ \text{iter} \right] T
\]

where: \( E \) is the elastic modulus; \( A \) is the member’s area; \( L \) is the member’s length; \( J \) is the modulus of inertia and \( T \) is the member’s axial load. The geometric stiffness matrix is updated, at each iteration, using the value of the member’s axial load \( T \) at the previous step. The convergence tolerance of the non-linear iterations can be assigned as a suitable percentage of the maximum axial loads.

4. Numerical application

4.1. Description of the case study

The previously described procedure has been applied to optimize the configuration of a guyed broadcasting antenna for telecommunications or mobile-phone networks, subjected to turbulent wind loading.

The structure is a 33 m high cable-stayed mast whose main pole is built with a steel hollow circular cross-section, having outer diameter \( D = 16.83 \text{ cm} \) and area \( A = 58.8 \text{ cm}^2 \), constant along the height, and supported by two orders of stays having diameter \( D = 2.7 \text{ cm} \) (Fig. 2).

The same structure was the object of a previous experimental study in a wind tunnel [13] aimed at investigating the aerodynamic loads acting upon it and the directionality effects of the wind loading.

Without any restriction to the applicability of the procedure, a schematic representation of the structure in two dimensions is considered in the analyses.

The finite-element model of the structure along with the joints numbering is shown in Fig. 2. The pole and the cables are represented using frame and truss elements, respectively. The stays’ pretension is taken into account by means of proper thermal loads applied to the relevant members.
4.2. The optimization problem

It is well known that the main constraints to the design of broadcasting antennas are the need of satisfying severe deformability limits in order to allow the signals to be properly transmitted and the requirement of restraining the overall dimensions in plan to reduce the cost for the terrain occupancy.

Thus, two proper indicators of the above restrictions have been chosen as components of the multiple objective function: the first one, \( f_1(x) \), is the sum of the squares of the displacements \( \delta_i \) and the second one, \( f_2(x) \), is the width in plan of the structure.

The design variables are the positions of the cable anchorages along the pole, \( h_1 \) and \( h_2 \), and the distance \( d \) between the cable anchorages to the ground (Fig. 2).

The problem's constraints are represented by the fulfillment of the safety and serviceability requirements.

Therefore, the optimization problem is expressed as follows:

\[
\text{minimize}_{x \in \Omega} f(x)
\]

where: \( x = (h_1, h_2, d) \);

\[
f(x) = f_1(x) + \alpha f_2(x) = \sum \delta_i^2(x) + \alpha \cdot d(x);
\]

\[
\Omega = \{ x : |\sigma_{\text{members}}(x)| \leq \bar{\sigma} \cap |\sigma_{\text{stays}}(x) \geq 0 \}.
\]

The feasible space \( \Omega \) is formed by the set of the design variables leading to internal stresses in each structural member satisfying pre-established safety requirements. In particular, the cables’ prestress must be greater than zero and the internal stresses in the elements of the mast, due to both the axial force \( N \) and bending moment \( M \), cannot exceed the allowable value \( \bar{\sigma} \), satisfying the inequality:

\[
\sigma = \left\| \frac{N}{A} \pm \frac{M}{W} \right\| \leq \bar{\sigma} = 240 \text{ N/mm}^2.
\]

A preliminary calibration of the cooling parameters used in the optimization procedure was performed in order to find the cooling schedule providing the best compromise between algorithm efficiency and computational effort. As a result, the control parameter \( T \) is reduced by 1% in each external loop, starting from an initial value \( T = 100 \), until the final value \( T = 10^{-3} \) is reached. The maximum number of the accepted configurations \( N \) was set to 150.

The shape of the objective function domain has been investigated to verify the convexity of the feasible design space, a property that allows the use of the simple weighting coefficients method. For this purpose, the components of the objective function have been calculated for a large number of design variables sets. In particular, the design variables \( h_1, h_2 \) have been varied within the range 5–33 m with 30 cm intervals and \( d \) has been varied from 2 to 6 m with 30 cm intervals. The components of the objective function \( f_1(x) \) and \( f_2(x) \), provided that the design variables \( x \) satisfy the constraints, are represented in Fig. 3. It can be seen that the feasible design space, resulting from the parametric analysis and represented in the plane \( f_1(x), f_2(x) \), is convex by the side where the optimal solution is expected. The convexity of the feasible design space allows all the possible Pareto-optimal solutions of the problem under investigation to be identified.

The ratios \( \alpha \) between the weighting coefficients \( \alpha_1 \) and \( \alpha_2 \) in Eq. (2) have been chosen to accommodate the differences in the physical dimensions of the components \( f_1(x) \) and \( f_2(x) \) and to make \( f_1(x) \) and \( \alpha f_1(x) \) comparable in magnitude. With this aim, a parametric analysis to evaluate the variability of \( f_1(x) \) and \( \alpha f_2(x) \) with the ratio \( \alpha \) was performed. Fig. 4, where the results are represented, shows that the contributions of \( f_1(x) \) and \( f_2(x) \) to the objective function \( f(x) \) are equivalent if the ratio \( \alpha \) is around 0.28. In the present study the ratio \( \alpha = 0.5 \) was chosen to enhance the role of the displacements in the optimization process. As \( \alpha_1 \) was set to unity, \( \alpha_2 \) was taken as 2.

The components of the objective function have been normalized to the same origin by dividing each term by the value it takes in the first guess configuration.

4.3. Results

The optimization problem is solved by considering several structural analysis techniques: static analysis under equivalent...
loads, dynamic analysis in the frequency domain (FD) and non-linear dynamic analysis in the time domain (TD).

The drag coefficients used to compute the wind loads are those obtained by wind tunnel tests on the same structure [13]. For the definition of the atmospheric boundary layer, according to Eurocode 1 [14], the following parameters have been chosen: $K_r = 0.19$, $z_0 = 0.05$ and $V_{\text{ref}} = 27$ m/s. For the static analysis, the gust factor suggested by Eurocode 1 is used.

In the case of the dynamic analysis in the frequency domain, the structural response is evaluated through the superposition of the first four natural modes, that proved to give a non-negligible contribution in a preliminary spectral analysis.

The stability of the solution with respect to the choice of the first guess set of design variables has been investigated by considering both the equivalent static analysis and the dynamic analysis of the response. For this purpose the dynamic analysis has been performed using the frequency domain approach to reduce the computational effort. The results in Table 1 show that the algorithm is reasonably independent from the first guess configuration. However, the accuracy could be improved by increasing the number of loops, i.e. refining the cooling schedule.

Since both the cables’ anchorages heights and widths are allowed to vary, the algorithm identifies different sets of optimal configurations, all equally effective. The small discrepancy between the values of the objective function may be ascribed to numerical tolerances and rounding. To study this problem in detail a parametric analysis was performed oriented to the systematic investigation of the feasible designs. Fig. 5 shows that the objective function has a unique minimum in a low gradient that can lead to solutions slightly different from each other.

The results in Table 1 highlight the discrepancy between the optimal design solutions obtained with the static analysis and those computed with the dynamic analysis. One of the reasons for this behavior is that the static analysis is based on the definition of equivalent static loads, which are calibrated on the first modal response of a free-standing cantilever beam. It is clear that this type of load fails to simulate accurately the dynamic response of cable stayed masts, whose first modal shape considerably departs from that of a cantilever and displays inversion points [15]. Moreover, for this class of structures, the contribution of the higher modes may hardly be neglected, as pointed out, among others, by Davenport and Sparling [16]. Obviously only the use of a non-linear step-by-step time domain analysis removes the need of any simplifying assumptions.

Table 1

<table>
<thead>
<tr>
<th>First guess design variables (m)</th>
<th>Static analysis</th>
<th>Dynamic analysis in the FD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal design variables (m)</td>
<td>Objective function</td>
<td>Optimal design variables (m)</td>
</tr>
<tr>
<td>21.00; 31.00; 6.00</td>
<td>18.462; 31.595; 3.369</td>
<td>1.977</td>
</tr>
<tr>
<td>19.00; 30.00; 5.00</td>
<td>19.262; 31.742; 3.302</td>
<td>1.975</td>
</tr>
<tr>
<td>22.00; 29.00; 4.00</td>
<td>18.772; 31.568; 3.354</td>
<td>1.972</td>
</tr>
<tr>
<td>20.00; 30.50; 3.00</td>
<td>19.059; 31.469; 3.328</td>
<td>1.973</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

To demonstrate the applicability of the non-linear structural analysis in the optimization procedure, both time domain (TD) and frequency domain (FD) optimizations were carried out.

Since the computation time required to perform a step-by-step analysis is about 100 times higher than the time needed for a frequency domain analysis, the comparison between the two procedures was carried out by considering a short duration of the loading, corresponding to 60 s, thought sufficient to excite the structure’s dynamic response.

As the main purpose of the numerical applications is to illustrate the proposed procedure from a methodological point of view, a single run of the TD optimization was carried out instead of the large number required by the Monte Carlo simulation which should be performed to take into account properly the stochastic nature of the applied load.

When using the TD approach the mean wind load was increased progressively from zero to the actual mean value during the first 5 s of the time histories, in order to avoid the sudden application of loads at their full value, which would have altered the structural response. The amplitude of each time step used in the analyses was set to 0.01 s. The convergence for the non-linear iterations was checked by accepting an unbalanced axial load of 1%.

In Table 2 are summarized the optimal configurations and the values of the objective function obtained using both the frequency and time domain analyses. The optimization in the time domain yields to a further reduction of the objective function with respect to the frequency domain analysis. The corresponding results in terms of internal forces at the optimized configurations are shown in Table 3. The higher
Table 2
Optimal configuration \((h_1, h_2, d)\) and values of the objective function obtained with the proposed procedure using dynamic analyses in the frequency and time domain

<table>
<thead>
<tr>
<th>First guess design variables (m)</th>
<th>Dynamic analysis in FD</th>
<th>Dynamic analysis in the TD</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.00; 31.00; 6.00</td>
<td>20.04; 30.63; 5.82</td>
<td>21.79; 29.04; 5.06</td>
</tr>
</tbody>
</table>

Table 3
Internal forces in correspondence of the optimized configurations obtained with the dynamic analyses in the frequency and time domain

<table>
<thead>
<tr>
<th></th>
<th>Dynamic analysis in the FD</th>
<th>Dynamic analysis in the TD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum cables' axial load</td>
<td>(11.35 \times 10^4)</td>
<td>(9.65 \times 10^4)</td>
</tr>
<tr>
<td>Maximum pole's bending moment</td>
<td>(3.12 \times 10^4)</td>
<td>(1.18 \times 10^4)</td>
</tr>
</tbody>
</table>

values of the internal forces obtained with the FD analysis with respect to the TD analysis are mainly due to the rule used to superimpose the modal responses (SRSS), which systematically overestimates the response of structures where modes with inversion points give a relevant contribution. This overestimation of the displacements exceeds the geometric non-linear effects which are taken into account properly by the time domain optimization.

5. Concluding remarks

A numerical procedure for the optimization of flexible wind-excited structures is proposed. It is based on the Simulated Annealing algorithm that, under proper hypotheses on the rate of decrease of the control parameter, converges to the global minimum.

The procedure is applicable to multiple objective problems and the structural response can be evaluated using several analysis methods.

The dynamic analysis can be performed both in the frequency and in the time domain. The latter type of analysis, carried out through the direct integration of the equation of motion, allows us to consider the geometric non-linearity, that can be significant for problems involving flexible structures.

The time domain procedure has a wider field of applicability, but requires a significant computational effort. In contrast, the use of the frequency domain procedure is faster and represents a preliminary design tool whenever non-linear effects are negligible or attention must be paid to serviceability requirements.

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